Risk Estimation of Extreme Windstorms

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1. Introduction

Estimating the risk of extreme natural hazards has been a major issue in recent decades, but has been limited to the exploitation of historical catalogs, which usually do not exceed 40 to 50 years, and to numerical models, which require heavy computation while being unreliable for extrapolation above observed intensities.

Extreme Value Theory provides a theoretical framework to describe and model tails of statistical distributions. This poster describes a statistical procedure to learn the extremal behavior of rare events in order to build models that can quantify the recurrence of past events as well as safely extrapolate above observed intensity levels. With this methodology, we develop a stochastic weather generator for extreme windstorms over Europe.

2. Univariate extreme value theory

Under mild conditions on a random variable X, the central limit theorem gives

$$n^{1/2}\left\{\left(n^{-1}\sum_{i=1}^{n} X_i\right) - \mu\right\} \to N(0,\sigma^2), \quad n \to \infty,$$

with $\mu \in \mathbb{R}, \sigma > 0$, and thus normal distribution is a universal approximation of $\sum_{i=1}^{n} X_i$ up to an affine rescaling. The generalized Pareto distribution

$$H_{\xi,\sigma}(x) = \begin{cases} (1+\xi x/\sigma)_+^{-1/\xi}, & \xi \neq 0, \\ \exp(-x/\sigma), & \xi = 0. \end{cases}$$

where $\sigma > 0$ and $a_+ = \max(a, 0)$ has a similar role in approximating tail distributions. Indeed, for a sequence of increasing theshold u_n,

$$\Pr(X - u_n > x \mid X > u_n) \to \begin{cases} H_{\xi,\sigma}(x) \\ 0, \end{cases}, \quad n \to \infty. \qquad (1)$$

The tail index ξ determines the regime of tail decay:

- $\xi > 0$: Fréchet type with $x \geq \mu$ and polynomial tail decay,
- $\xi = 0$: Gumbel type with $x \geq \mu$ and exponential tail decay,
- $\xi < 0$: Weilbull type with bounded tail, $x \in (u, u - \sigma/\xi)$.

Since the generalized Pareto distribution is the only possible limit distribution for threshold exceedances, for any random variable X and large enough threshold $u < \inf\{x : F(x) = 1\}$, the approximation

$$\Pr(X - u > x \mid X > u) \approx H_{\xi,\sigma}(x).$$

provides a model for tails, which is, due to (1), safe for extrapolation above observed intensities. However, severe climatic events, such as floods, windstorms, heatwaves, cannot be modelled using only univariate extreme value theory, as it fails to capture their spatio-temporal nature.

3. Functional peaks-over-threshold analysis

3.1 Functional exceedance

Characterisation of an exceedance for a univariate quantity is straightforward: for some threshold $u > 0$, any observation X such that $X \geqslant u$ is defined as an exceedance. When X is a function the definition of an exceedance can be less intuitive.

If $X \in C(S)$ is a continuous function over a compact subset $S \subset \mathbb{R}^d$, we define an exceedance as an event $\{\mathrm{r}(X) \geqslant u\}$, where r is a monotonic increasing functional, called a 'risk functional'. Common examples are

- $\sup_{s \in S} X(s)$ for events where X exceeds a threshold at least at one location;
- $\sum_{t=1}^{T} \int_S X_t(s)ds$ for spatio-temporal accumulation;
- $\sqrt{\int_S X(s)^2 ds}$ as a proxy of the energy inside a system;
- $X(s_0)$, for risks at a specific location $s_0 \in S$.

3.2 r-Pareto process

A generalized r-Pareto process P (de Fondeville and Davison, 2018a) is defined by

$$P = \begin{cases} R\frac{W}{\mathrm{r}(W)}, & \xi \neq 0, \\ R + \log W - \mathrm{r}(\log W), & \xi = 0, \end{cases}$$

where

- R, called the radial component, is univariate generalized Pareto with tail parameter ξ, and distribution function

$$\Pr(R > \rho) = \left(1 + \xi\frac{\rho - u}{\sigma}\right)^{-1/\xi}, \quad \rho \geqslant u \geqslant 0,$$

  with $\sigma > 0$. R determines the intensity of the r-exceedance;
- W, the angular component, is a stochastic process on the unit sphere $\{x \in C(S) : \|x\|_1 = 1\}$ with probability measure σ_{ang}, which determines the dependence of P.
- The r-exceedance distribution of P is

$$\Pr\{\mathrm{r}(P) \geqslant \rho\} = \left(1 + \xi\frac{\rho - u}{\sigma}\right)^{-1/\xi}, \quad \rho \geqslant u.$$

- The generalized r-Pareto process has generalized Pareto marignals above a sufficiently high threshold $u_0 \geqslant 0$:

$$\Pr\{P(s_0) \geqslant \rho \mid P(s_0) \geqslant u_0\} = \left\{1 + \xi\frac{\rho - \mu(s_0)}{\sigma(u_0)}\right\}^{-1/\xi}, \quad \rho \geqslant u_0,$$

  with $\sigma(u_0) > 0$ and $\mu(s_0) \in \mathbb{R}$

3.3 Convergence

The r-Pareto process is the only possible limit of rescaled threshold exceedances, i.e.,

$$\Pr(X - u_n \in \cdot > x \mid \mathrm{r}(X) > u_n) \to \begin{cases} \Pr(P \in \cdot), \\ 0, \end{cases}, \quad n \to \infty. \qquad (2)$$

and thus for a stochastic process X and large enough threshold $u > 0$,

$$\Pr(X \in A) \approx \Pr\{\mathrm{r}(X) > u\} \times \Pr(P \in \cdot), \qquad (3)$$

with $A \subset \{x \in C(S) : \mathrm{r}(x) \geqslant u\}$.

Consequently the generalized r-Pareto process models the tail of any stochastic process X and the convergence (2) ensures safe extrapolation above observed intensities. In equation (3), $\Pr\{\mathrm{r}(X) > u\}$ represents the overall probability of observing an extreme event, while the part relative to the r-Pareto process describes the intensity and pattern of the r-exceedances.

4. Statistical modelling

For a threshold $u > 0$ and risk functional r, the density function f_θ^{r} of the generalized r-Pareto process is

$$f_\theta^{\mathrm{r}}(x) = \frac{\lambda_\theta^{\mathrm{r}}(x)}{\Lambda_\theta\{A_{\mathrm{r}}(u)\}}, \quad x \in A_{\mathrm{r}}(u),$$

with $A_{\mathrm{r}}(u) = \{x \in C(S) : \mathrm{r}(x) \geqslant u\}$ and where

$$\Lambda_\theta\{A_{\mathrm{r}}(u)\} = \int_{A_{\mathrm{r}}(u)} \lambda_\theta^{\mathrm{r}}(x)dx, \quad u \geqslant 0.$$

The measure Λ must satisfy properties such as $\Lambda(tA) = t^{-1}\Lambda(A)$ to properly define a generalized r-Pareto process.

For environmental risk assessment, the vector x is usually high-dimensional, and the multivariate integral quickly becomes intractable. Also, because the generalized r-Pareto is an asymptotic model fitted to sub-asymptotic data, the estimation procedure must be robust to model misspecification.

- An adaptation of the gradient scoring rule (de Fondeville and Davison, 2018b) allows statistical inference using partial derivatives of the log-density function with respect to $x_1, \dots, x_\ell$,

$$\delta_{\mathrm{w}}(\lambda_{\theta,u}^{\mathrm{r}}, x) = \sum_{i=1}^{\ell}\left(2w_i(x)\frac{\partial w_i(x)}{\partial x_i}\frac{\partial \log \lambda_{\theta,u}^{\mathrm{r}}(x)}{\partial x_i} + w_i(x)^2\left[\frac{\partial^2 \log \lambda_{\theta,u}^{\mathrm{r}}(x)}{\partial x_i^2} + \frac{1}{2}\left\{\frac{\partial \log \lambda_{\theta,u}^{\mathrm{r}}(x)}{\partial x_i}\right\}^2\right]\right),$$

  where w is a differentiable weighting function, that vanishes on the boundaries of $A_{\mathrm{r}}(u)$.
- Minimization of the loss function δ_{w} gives an asymptotically unbiased and normal estimator of the model parameters θ, that is robust and computationally cheap.

5. Application to extreme windstorm in Europe

We apply the previous results to extreme windstorms over Europe. To do so, we study 3s maximum windgusts measured every 3 hours over the period 1979 to 2016 using data from ERA-Interim reanalysis (Dee et al., 2011).

5.1 Storm definition

For insurers and regulators, the most damaging windstorms impact areas with dense human infrastructure. Thus, we define a storm as an exceedance of the 24-hour temporal maxima of the spatial mean in a small region including Paris, London, Brussels and Amsterdam, as shown in Figure 1, i.e.,

$$\mathrm{r}(X) = \max_{i=1,\dots,9} |S_{\mathrm{PLBA}}|^{-1}\int_{S_{\mathrm{PLBA}}} X(s)ds.$$

To suppress the effect of temporal clustering, we center the time frame on the maxima and keep only events that are at least 48 hours apart.

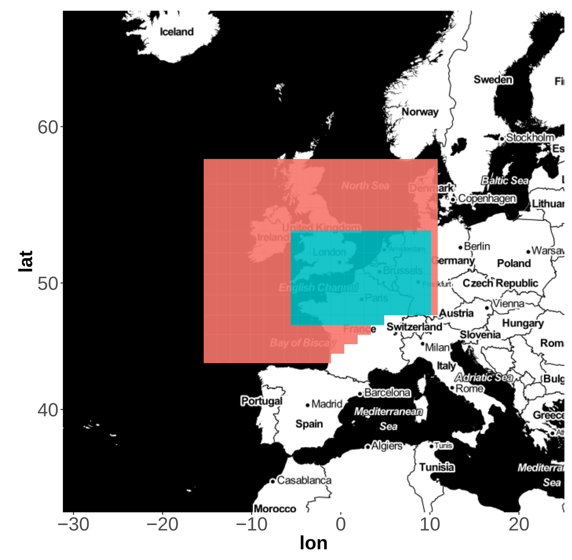

Figure 1: *Area of study (colored cells) for modelling extreme windstorms over Europe. Green cells represent the region with high density of human infrastructure.*

5.2 Windstorm frequency

We set the threshold $u > 0$ to obtain an overall total of 117 r-exceedances, yielding about 3 storms per year. As a model for $\Pr\{\mathrm{r}(x) > u\}$, we use a logistic regression with the North Atlantic Oscillation and temperature anomaly as covariates; both were found to be highly significant. See Figure 2.

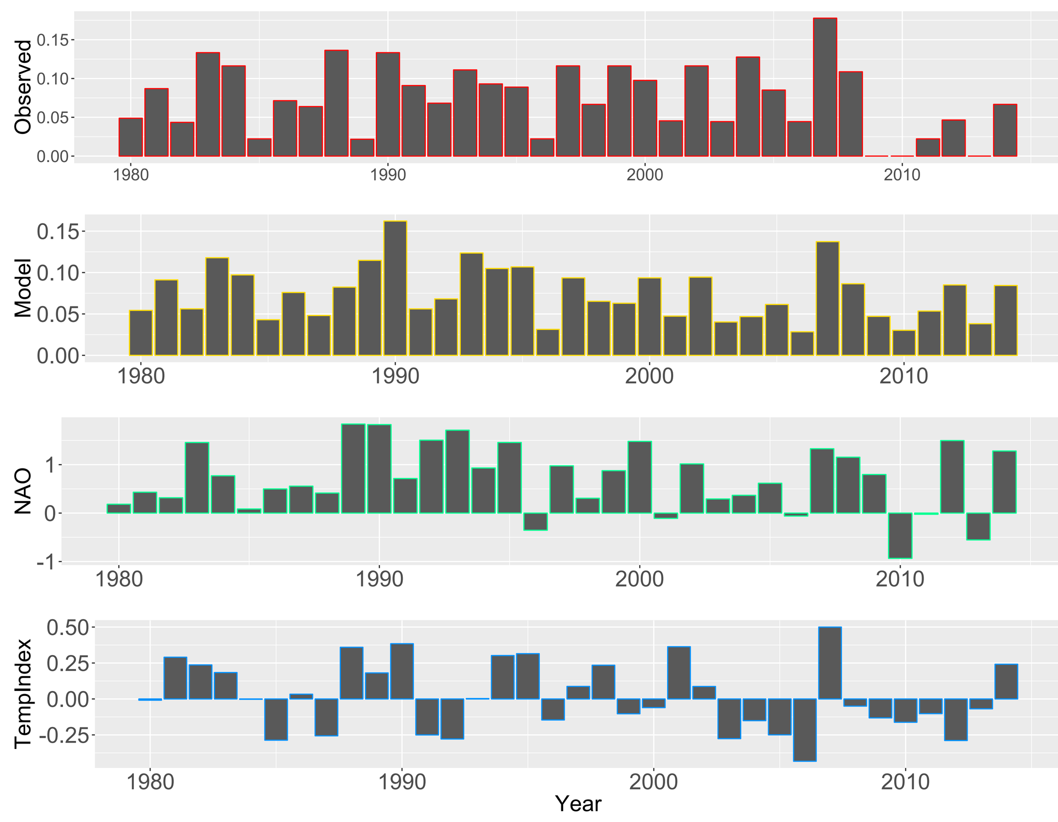

Figure 2: *Model for the annual probability of storm occurrence. Observed frequency (Top), modelled frequency (Second row), mean North Atlantic Oscillation index (Third row), temperature anomaly index (Bottom).*

5.3 Storm model

For the radial component W, we use a log-Gaussian process because classical covariance models developed in spatial-temporal statistics for Gaussian processes, can be used to parametrize the dependence of the Pareto family.
Figure 3 shows a simulation of a storm with an intensity equivalent to storm Lothar, which occurred during the winter 1999–2000, for which the estimated insured loss is around 8 billion dollars. Our model estimates that we should expect a storm like Lothar once every 19 storms, on average.

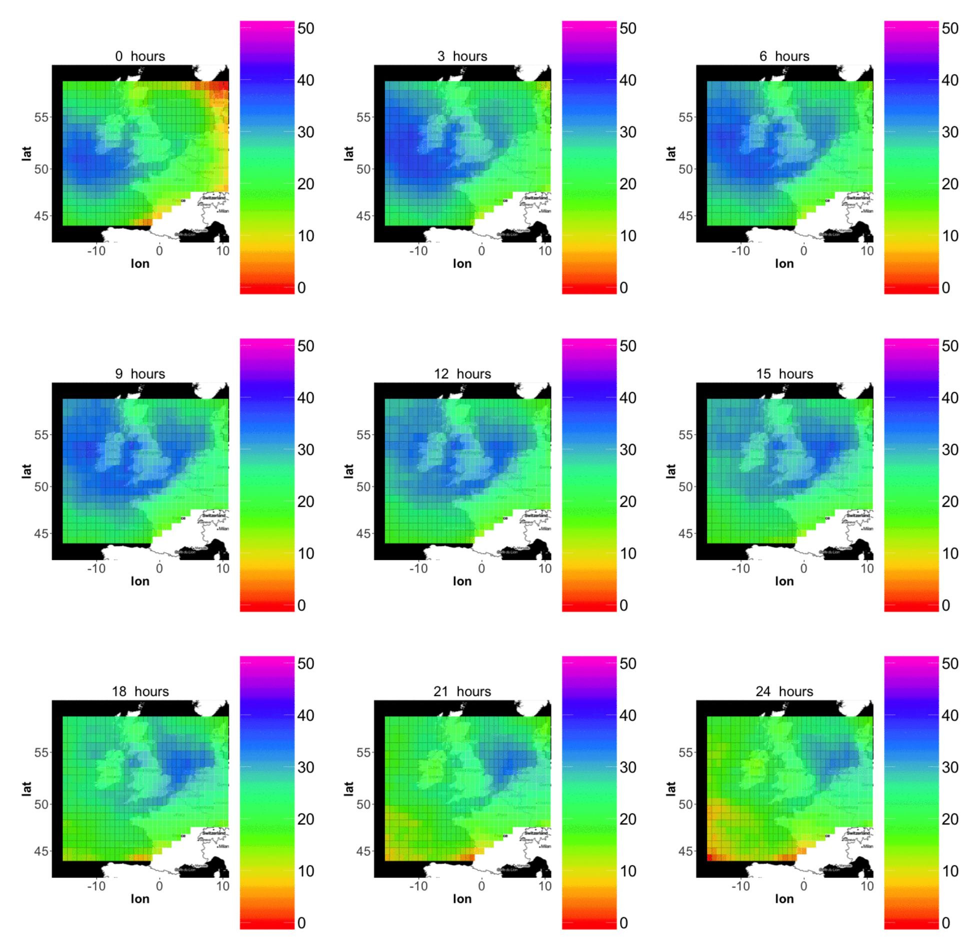

Figure 3: *Simulated windstorm with an intensity equivalent to storm Lothar, which occurred during the winter 1999.*

6. Acknowledgement

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